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# Energy Demand Models and Modelling


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**Executive Summary**

One of the key interests of energy demand analysts is in determining the responses of consumers to changes in energy prices, typically as reflected in estimated price elasticities. Since these price elasticities cannot be directly observed from the data, one of the major problems facing such analysts concerns the choice of model that is to be used to estimate these measures. This turns out not to be a simple decision because of the way in which energy demand models have evolved, and the fact that no single model has been shown to subsume or dominate all the others. As a result, analysts have often tended to choose the simplest models, although without necessarily considering whether the empirical evidence supports this decision, and hence whether the resulting estimates will be useful. Since these estimates are often used to evaluate the effects on consumer behaviour of various policy initiatives – for example, any type of tax would typically be expected to affect prices – the use of inappropriate models could have far-reaching consequences. The purpose of this paper is to explain energy demand models and modelling in the context of the historical evolution of the approaches that have been used. Limiting attention to non-transport energy demand, and emphasizing model development and specification, examples are used to illustrate the different approaches, explain the types of information each yields, and outline how they differ and their advantages and disadvantages.

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## 1. Introduction

One of the consequences of the first world oil shock in the early 1970s was a marked increase in energy demand analysis, and specifically in modeling energy demand patterns. There appear to be at least four primary motivations for developing and estimating such models. First, there is the question of the magnitude of demand responses as a result of price changes and income changes that may occur. Clearly these responses can have important implications for policy analysis, since any type of tax would generally raise the price and hence affect demand, as would increases in income, perhaps as a country develops. Second, there is a general interest in forecasting or predicting future energy needs, and such forecasts are generally anchored in knowledge of what has happened in the past, how past demand behaviour depends on various factors, and expectations about how those factors might change in the future. Third, while energy seldom plays the same role in economic analysis as labour and capital, there is a general understanding that without energy there would be no production, so that issues of the extent to which energy can be substituted in the production (or other) process has become an important consideration. Fourth, with increasing concern over greenhouse gas (GHG) emissions and global warming, which tend to be largely associated with energy production and consumption, questions of how demand for energy can be curtailed, or converted to forms associated with fewer emissions have received increased prominence.

Driven by such considerations as computing power, data availability, and even the background and training of the original analysts, early attempts at modeling energy demand were, at least in today's terms, relatively simplistic. However, all these factors have evolved over time, and as a result energy demand models and modeling have changed considerably over the past three to four decades, although perhaps somewhat surprisingly, versions of those simplistic early models are still found in many recent energy demand modeling exercises.

The purpose of this paper is to review energy demand models and modeling, with a view to explaining the key features of the models and estimation methodology – which tend to be inextricably linked – as well as how these have evolved over time and, where possible, suggesting at least partial explanations for this evolution. To do this, the models are considered in the context of the data that were available at particular times, and the extent to which previous models might have appeared to be no longer adequate as more data accumulated, technology improved, and the questions of interest evolved.

A point to emphasize at the outset of our analysis is that there is no single “right” approach to modeling energy demand. Models differ according to various circumstances, and the model that might be applicable in one setting may be totally inappropriate in another. While it is not possible in a limited space to consider the many variations of model specifications that have been developed by different authors, our aim is to provide a feel for the types of models that have been considered, and the reasons for their development. In several cases, particular approaches have just seemed to end, without a specific replacement, perhaps because they could not be developed any further, or possibly just because interest in the questions that they were designed to answer waned. It certainly does not appear to be the case that most of those models could yield no further insights into energy demand, so perhaps they may be resurrected at some point in the future.

The plan of the remainder of this paper is as follows. In Section 2 we consider early approaches to model specification and estimation, particularly the focus on a single fuel and the functional specification in which all variables are in (natural) logarithms. Systems of equations approaches to modeling energy demand, both at the macroeconomic and microeconomic level, are reviewed in Section 3. In Section 4 we address the issue of the implications for energy demand modeling and estimation of the potential non-stationarity of many of the relevant data series. The possibility of asymmetric demand responses to energy price changes, and issues associated with the specification and estimation of models that allow such behaviour are considered in Section 5. Section 6 contains a brief introduction and review of other aspects of energy demand models and modeling, while Section 7 summarizes and concludes.

## 2. Early approaches to modelling and estimation

The earliest specifications of energy demand equations appear to have had the general form:

(1) $\ln E = \beta_1 + \beta_2 \ln P + \beta_3 \ln Y + e$

where $E$ represents energy consumption, that is the quantity of energy consumed, or possibly the quantity consumed of a particular energy source such as oil, $P$ represents its price, $Y$ represents a measure of income or aggregate economic activity, $e$ is a stochastic error term, and the $\beta_j$'s are unknown parameters. By specifying the model in this way, with all variables in natural logarithms, (the so-called log-linear, or linear in logs, or double-log functional form) the coefficients are readily interpreted as elasticities, so that, for example $\beta_2$ is the (own) price elasticity:

(2) $\beta_2 = \partial \ln E / \partial \ln P = (\partial E / \partial P)(P / E) = (\partial E / E)/(\partial P / P)$,

which indicates the proportional response in energy demand sue to a proportional change in price holding constant the values of all other factors specified in the equation (in this case income). Of course convenience of interpretation is not a particularly good reason for specifying the model in logarithms, and ideally the functional form would be evaluated against other alternatives, as was considered later in several studies (for example, Chang and Hsing, 1991). However, in fairness, estimation of nonlinear functions that nest linear and log-linear specifications required increased computing power, and in these early approaches, little attention was paid to evaluating alternative functional specifications using model diagnostics, such as autocorrelation in the error term that might exist with one particular function but not with another. In addition, if all the variables in (1) were in linear form rather than natural logarithms, the price elasticity would be obtained as $\beta_2(P / E)$, and therefore would no longer be constant over the sample. Although the elasticity in this case could readily be evaluated using the sample means, or at some other point, or presented for a range of values of P and E, such an approach was not widely adopted. Also, while the estimated standard error of the estimated parameter on $\ln P$ in the log-linear function (which we denote by $\hat{\beta}_2$), $\hat{se}(\hat{\beta}_2)$ yields an estimate of the standard error of the price elasticity, in a linear specification this would not be the case, so that unless both E and P are treated as non-random, in which case the standard error of the price elasticity would be given by $\hat{se}(\hat{\beta}_2)(P / E)$, determination of an estimated standard error for the elasticity (using the estimated E in place of actual data) would require nonlinear methods (such as the delta method defined in Greene, 2008:68) that were not as easily implemented in the 1970s and early 1980s.

Apart from these computational issues, there are other drawbacks of log-linear functional forms, including their generally not being consistent with optimizing behaviour. In some cases authors attempted to rectify the constancy of the elasticity by altering the model specification (Dias-Bandaranaike and Munasinghe, 1983) but generally such ad hoc approaches only serve to introduce additional problems (Plourde and Ryan, 1985). Perhaps the most problematical aspect of the specification in (1) however, was not the functional form, but the lack of any dynamic structure. Many authors, within an energy context and in other circumstances, have noted the need to allow for long-run responses to differ from short-run responses (for example, Berndt et al, 1981). To the extent that the capital stock in place requires energy to operate, or requires a certain source of energy, the substitution that might be expected due to an energy price increase first requires a change in the capital stock, and such changes often cannot be enacted instantaneously. In such circumstances, the long-run response to a price increase would be expected to exceed the short-run response, since in the long run greater substitution possibilities will be available as the capital stock is changed. Again, empirical evidence of the need for such dynamics may have shown up in the model diagnostics – in terms of evidence that the errors were autocorrelated – had such diagnostics been obtained. The simplest dynamic specification, which Berndt et al (1981) and others refer to as first-generation models, simply include a lagged dependent variable as an explanatory variable. Thus, with time subscripts added, the model specification in (1) is replaced with:

(3) $$\ln E_t = \gamma_1 + \gamma_2 \ln P_t + \gamma_3 \ln Y_t + \gamma_4 \ln E_{t-1} + e_t$$

where $E_{t-1}$ is energy consumption in the previous period, and the unknown parameters are now represented by $\gamma_j$'s, for reasons that will become apparent below.

With the specification in (3), the effect of a price (or income change) is now different in the short-run and in the long-run. Specifically, in the short-run, the price elasticity would be $\gamma_2$, the coefficient on $\ln P_t$ as before. Typically, the long run is defined as the period sufficiently long to have enabled all adjustments to have taken place, so that $E = E_t = E_{t-1}\ (= E_{t-2} = E_{t-3} = \cdots)$. Substituting this equality in (3), grouping terms, and then taking the derivative, the long-run price elasticity is given by $\partial \ln E / \partial \ln P = \gamma_2 / (1 - \gamma_4)$. In this case there is no avoiding the use of nonlinear methods, such as the delta method, to obtain an estimated standard error for the estimated elasticity. Typically these standard errors were simply not calculated, and it was simply noted that the long-run and short-run elasticities were different, with the magnitude of the (negative) long-run value typically exceeding the magnitude of the (negative) short-run value, provided $0 < \gamma_4 < 1$.

One of the main criticisms levelled at the model specification in (3) was that it was purely *ad hoc* in nature. Indeed, the so-called second- and third-generation models that we consider later were developed largely in response to this criticism. Despite this criticism, there are a number of possible justifications – or perhaps rationalizations – for the introduction of the lagged dependent variable in (3). These include a partial adjustment or stock adjustment model, whereby, perhaps due to the need to introduce different capital equipment, a desired change in energy consumption could not be satisfied in the current period. Thus, in (1), the dependent variable $\ln E_t$ would be replaced with desired energy consumption, $\ln E_t^*$, so that this equation would now model how *desired* energy consumption responds to changes in price and income, and the coefficients in this model, the $\beta_j$'s, would be interpreted as indicating the long-run responses of energy demand to changes in the explanatory factors. However, due to technological constraints, actual consumption could only partially adapt to this desired level via the relationship:

(4) $$\ln E_t - \ln E_{t-1} = \theta(\ln E_t^* - \ln E_{t-1}), \quad 0 \le \theta \le 1,$$

where the parameter $\theta$ represents the speed of adjustment, with $\theta = 0$ indicating no adjustment in the current period, $\theta = 1$ indicating instantaneous adjustment, and values between these two extremes indicating partial adjustment. Substituting (1), which now represents $\ln E_t^*$, into (4) and rearranging yields the model in (3), where $\gamma_j = \theta\beta_j$, $j$ = 1, 2, 3, and $\gamma_4 = (1 - \theta)$, and therefore $\gamma_j / (1 - \gamma_4) = \beta_j$, the long-run effects.

Of course, this is not the only possible way to rationalize the lagged dependent variable that appears in (3). For example, in (1), the terms $\beta_2 \ln P_t$ and $\beta_3 \ln Y_t$ could be replaced with geometric lags, that is an infinite distributed lag where the coefficients decline geometrically. Thus, for example, in (1), $\beta_2 \ln P_t$ would be replaced with $\beta_2(\ln P_t + \lambda \ln P_{t-1} + \lambda^2 \ln P_{t-2} + \lambda^3 \ln P_{t-3} + ...)$, where $0 < \lambda < 1$, while $\beta_3 \ln Y_t$ would be replaced with $\beta_3(\ln Y_t + \lambda \ln Y_{t-1} + \lambda^2 \ln Y_{t-2} + \lambda^3 \ln Y_{t-3} + ...)$, so that $\beta_2$ and $\beta_3$ would represent short-run effects of changes in P and Y, respectively, while the corresponding long-run effects (when $P_t = P_{t-1} = P_{t-2} = \cdots$, and $Y_t = Y_{t-1} = Y_{t-2} = \cdots$) would be given by $\beta_2(1 + \lambda + \lambda^2 + \cdots) = \beta_2/(1-\lambda)$ and $\beta_3(1 + \lambda + \lambda^2 + \cdots) = \beta_3/(1-\lambda)$. Hence the effect of a price or income change has an immediate effect as well as an effect in each subsequent period that is always smaller than the effect in all preceding periods. Making these substitutions yields an equation that includes infinite lags on the explanatory variables, although relatively few parameters. To obtain an estimating equation a Koyck transformation is used. Specifically, we calculate ($\ln E_t - \lambda \ln E_{t-1}$), so that all the lag terms cancel, resulting in an estimating equation that has the same form as in (3) where $\gamma_1 = \beta_1(1-\lambda)$, $\gamma_j = \beta_j$, $j$ = 2, 3, and $\gamma_4 = \lambda$, and $\gamma_j/(1-\gamma_4) = \beta_j/(1-\lambda)$ would yield the long-run effects.

A different type of rationalization for the inclusion of lagged values of energy consumption, as well as possibly lagged values of the other explanatory variables, is outlined by Bentzen and Engsted (2001). Here the primary motivation concerns the properties of the estimators and their standard errors. As we discuss in more detail in Section 4, a problem with estimating models like (1) in a time series context is that if the variables are non-stationary (trending), the regressions may be spurious. However, if appropriate lags of all the variables are included in (3), and there is a unique long-run (cointegrating) relationship among the variables being studied, then the model written in levels form as in (3) remains valid and asymptotically-valid hypothesis testing can be conducted in the usual way.

Regardless of the rationalization that is used to justify the inclusion of the lagged dependent variable as an explanatory variable in (3), the modeling is *ad hoc*, with no justification for the lag structure based on any real consideration of economic behaviour. Additional criticisms of this approach were provided by Berndt, Morrison and Watkins (1981), hereinafter BMW. Generalizations of these dynamic structures to what BMW refer to as second and third generation models typically involve systems of equations rather than a single equation, so that the interrelationships between different inputs (such as labour, capital, energy and materials), or between different sources of energy, can be explicitly recognized. These types of models are considered in the next section.

While it may be attractive to think of the evolution of models – such as from those with first-generation dynamics to those with more sophisticated dynamic specifications – as occurring due to theoretical developments, if there was not some perceived problem with the application of the simpler models it would certainly seem less likely that the more sophisticated models would be developed and receive relatively widespread acceptance. After all, one of the most cherished aspects of modeling is parsimony, the ability to abstract from reality with models that are relatively simple and involve few parameters. Indeed, it could be argued that this criterion, parsimony, was one of the key factors resulting in widespread use of the log-linear specification, and one which maintains this simple specification as a workhorse of energy demand analysis even today. Therefore, it seems likely that there were some empirical problems that were perceived with the simpler specifications in (1) and (3) that resulted in the push to develop alternative specifications that were typically more complex.

## 3. Systems of equations approaches

Perhaps the most significant breakthrough in terms of econometric modelling in general, but particularly in modelling demand relationships, was the introduction of the Transcendental Logarithmic (translog) function by Christensen, Jorgenson, and Lau (1973). Until this point, energy demand analysis predominately involved a single-equation approach. The idea of a single aggregate function from which demand functions for individual goods or inputs could be derived and estimated was developed much earlier with the work of Stone (1954) and others. However, the aggregate function in these cases was usually a utility function, and the systems of demand equations referred predominately to complete descriptions of consumer expenditures. On the production side, there was only a limited set of production functions from which a set of input demand equations could be derived, particularly the Cobb-Douglas and Constant Elasticity of Substitution (CES) specifications. However, these were typically specified with two inputs – labour and capital – and were very restrictive. The main advantage of the translog form, like the many flexible functional form specifications that followed, was that it could approximate an arbitrary function to the second order, that is, it had sufficient parameters to avoid imposing restrictions on the first and second derivatives of the function, the first derivatives being the demands, and the second derivatives being the price effects, which are the major component of the price elasticities. In addition, it could readily be used in applications with almost any number of inputs or commodities.

In view of the different approaches and development of energy demand modelling of production and consumption, including the different issues that have arisen in each case, we examine these two areas separately.

### 3.1 Production-side modelling

Representations of energy demand have also been incorporated in models designed with broader purposes in mind, notably that of providing representations of the interaction between energy and the economy and for policy analysis purposes. Early on, two different approaches to dealing with energy demand within such models emerged. The "bottom up" approach arguably finds its roots in practices associated with the natural sciences and engineering. Since the focus here is on capturing the energy-using consequences of different technologies, detailed representations of the energy-use characteristics of different types of capital equipment are undertaken. By incorporating information on the prevalence of the various technologies, demand / consumption for each energy source at the sectoral level or for the economy as a whole (as appropriate given the modelling environment) can then be obtained by summing across technologies. On a similar note, summing across all available energy sources yields the total demand for energy by sector or for the economy as a whole. Schipper et al. (1985) provide an example of this kind of approach in their cross-country comparison of the drivers of energy use in the residential sector.

A different variant of this approach embeds the kind of rich technological detail described above in an optimization framework which, given an explicit objective function, can then be solved for the optimal approach to meeting the energy requirements of a sector or of the economy as a whole. Early versions of the MARKAL family of models are examples of this second variant of the "bottom up" approach to energy demand modelling.

The second type of approach to dealing with energy demand in broader-purpose models draws more heavily on economics and, as a result, will be discussed in more detail. A key characteristic of this approach is its treatment of energy as a factor of production within a representation of output production. The so-called "top-down" approach implemented in production-side models shied away from including much technological detail and instead focused on using economics-inspired relationships that were typically estimated econometrically using data for the relevant variables. Here, energy demand was determined through the production relationship and the demands for individual energy sources, to the extent that these were even considered, were typically determined by splitting total energy demand through econometrically estimated relationships.

Key early contributors in this area, such as Hudson and Jorgenson (1974), took advantage of the development of flexible functional forms for production functions, such as the translog. In its most common form, this approach views energy (E) as an input that is combined with physical capital (K), labour (L), and materials (M) to yield the output of a sector of the economy (e.g., manufacturing or sub-sectors of manufacturing) or of the economy as a whole. In some applications, materials are assumed to be part of what is modeled as being "produced" and thus only three factors of production are explicitly taken into consideration (e.g., Christodoulakis and Kalyvitis, 1997).

The standard approach begins with a representation of the production process:

$$y = g(K, L, E, M)$$

where y is a measure of real output and g(.) is a functional form for production, such as the translog production function. Some authors (e.g,. Chang 1994) have proceeded to estimate the parameters of the production function directly and thus obtain estimates of the elasticities of substitution among the various factors. However, by far the more common approach has been to start with a representation of production, invoke assumptions of cost-minimizing behaviour on the part of firms, and derive the dual cost function. Using this cost function, consistent factor demand equations can be derived and their parameters subsequently estimated.

The approach outlined above proved to be fertile grounds for the development and empirical implementation of flexible function forms (such as the translog). The early, influential work of Berndt and Wood (1975) is a clear example of a paper that both enhanced our appreciation of the usefulness of flexible functional forms in economic applications and furthered our understanding of the role of energy in the production process. Within a short period of time after this paper had appeared, a number of other contributions pushed further explorations of factor substitution possibilities by, among others, using alternative functional forms (e.g., Magnus 1979) and pooled data from a number of different countries (e.g., Griffin and Gregory 1976). While not universally obtained in these kinds of empirical applications, estimation results often suggested that energy and capital were complements in production. Field and Grebenstein (1980), for example, showed that the definition (and thus the measure) of "capital" used could influence the conclusion as to whether a complementary or substitutability relationship existed between capital and energy.

Fuss (1977) extended the work of Berndt and Wood (1975) by showing that if certain separability conditions are invoked, it is possible to apply a two-stage optimization approach and obtain not only a representation of factor demands (including energy), but also consistent expressions of the demands for individual energy sources or fuels. Fuss shows, however, that this extension comes at a cost: the separability conditions needed to support the two-stage budgeting approach also imply that there can be no level effects in the consistent representations of the demands for energy sources, and thus real output cannot appear in the individual share equations. A survey of the relevant literature indicates that this lesson has not been heeded in a number of contributions. Further details on weak separability and its implications are provided below in a consumer demand context.

Another direction explored in this literature has been to restrict the substitution possibilities across factors of production – and especially that between capital and energy - in the modelling efforts. This has typically involved the grouping of energy and capital as a "bundle" within the overall production representation:

$$y = f((K, E), L, M)$$

where, by construction, capital and energy would be substitutes within the bundle, and then complements within the overall representation of the structure of production. Berndt and Wood (1979) and Helliwell et al. (1987) are examples of contributions that adopted this kind of approach.

The notion of treating energy as a factor of production has also been overwhelmingly adopted in efforts to model entire economies. Today, many – if not most – macroeconometric models the world over incorporate a KLEM-type of approach to modelling aggregate (or sectoral) production. Similarly, numerous computable general equilibrium models also use a KLEM representation to incorporate energy as a distinct factor of production in production.

Efforts have also been made to reconcile the two approaches with a view of finding ways to include the relative strengths of both traditions within a single modelling framework. A number of variants of so-called "hybrid" models have been developed more recently and have drawn inspiration from an early effort in this direction by Hoffman and Jorgenson (1977). Basically, some bottom-up models have been modified to include richer economic detail, as has been the case with more recent versions of the MARKAL models (Schafer and Jacoby, 2006).

Efforts have also been directed at incorporating the technological detail characteristic of bottom-up approaches into top-down models. Bohringer (1998) provides an example of this extension within the context of a computable general equilibrium model, while Kohler et al. (2006) embeds technological detail in a framework more akin to a macroeconometric model.

Increases in computing power have served energy demand analysts very well by making it possible to estimate and solve increasingly complex representations of the economic and technological processes that underlie energy demand. Indeed, the recent hybrid models have improved our understanding of both the factors driving energy demand and the role of energy in the economy, thus yielding a greater ability to undertake meaningful energy policy analysis.

### 3.2 Consumer energy demand models and modelling

In addition to flexible functional forms, probably the key development that enhanced modeling and estimation of systems of equations for different types of energy, or different energy sources, was the empirical implementation of the assumption of weak separability. This assumption is a necessary and sufficient condition for two-stage budgeting. Consider, for example, aggregate energy demand which comprises demand for oil products, natural gas, and electricity (and possibly other products such as wood, propane, etc., which we omit here to simplify the analysis, but which can readily be included in particular applications where they are relevant). With two-stage budgeting, a consumer can be viewed as first determining the allocation of their budget to various aggregates – such as food, clothing, energy, etc. – and then for each of these aggregates, determining expenditures on the various items within that aggregate. At each stage, only certain variables are relevant to the decision-making. Thus, at the first stage, the consumer would require information on the total budget and the prices of each aggregate – the price of "food", the price of "clothing", the price of "energy", etc. Focusing on the energy group, at the second stage all that would be required is total expenditure on energy and the prices of each of the different types of energy that comprise the group. Thus, in specifying demands for individual types of energy, food prices, whether for the group as a whole or for individual food items, are not relevant, and neither are prices of or expenditures on any other good, or group of goods, outside of those contained in the group of energy products. Further, while both stages of the budget allocation process can be considered, there is no need to do so, and attention can be limited just to one group at the second stage, that is, in the context that is relevant here, just to the determination of demands for different sources of energy

Even with the development of the translog function and utilization of the weak separability assumption, there was one more key component in facilitating the specification and estimation of systems of energy demand equations. A difficulty with many utility function specifications, including the translog, is that the derived demand equations for any one good have the quantities of other goods as explanatory variables. Clearly, since utility is maximized by choosing the quantities subject to the budget constraint, these quantities are endogenous, resulting in difficulties in estimation. However, with the use of duality theory, it was realized that rather than specify a utility function that is maximized subject to a budget constraint, an equivalent representation was to minimize the cost of achieving a specified level of utility. This realization led to the specification of cost or expenditure functions, which depend only on prices and the level of utility. Using Shephard's lemma, demand equations can be obtained by differentiating the expenditure function with respect to a particular price.[1] The unobserved utility level is then substituted out using the relationship that the cost function must equal total expenditure. The translog cost or expenditure function was introduced by Christensen et al (1973) and, combined with the use of the weak separability assumption, revolutionized specification and estimation of energy demand equations.

Despite this breakthrough, demand equations derived from the translog suffered one major drawback in a consumer setting, namely the property of homotheticity, where the expenditure shares do not change as total expenditure changes. This drawback was not resolved until the expenditure or cost function corresponding to the Almost Ideal Demand System (AIDS) was introduced by Deaton and Muellbauer (1980). Since that time, the AIDS model (and several variants of it) has become the most common way to specify systems of consumer demand equations, including those for energy sources.

The AIDS model is based on the expenditure function:

[1] See, for example, Diewert (1976). Alternatively, the logarithmic form of Shephard's lemma can be used to derive the budget share equations as the derivative of the logarithm of expenditure with respect to the logarithm of price.

(5) $$\ln E_t\,(u_t, p_t) = \alpha_0 + \sum_j \alpha_j \ln p_{jt} + ½ \sum_j \sum_k \gamma_{ij} \ln p_{jt} \ln p_{kt} + u_t \beta_0 \prod_j p_{jt}^{\beta_j}$$

where $u_t$ is household utility in period $t$, $p_t = (p_{1t}, \dots, p_{nt})$ is the vector of prices prevailing in period $t$, and $\alpha_j$, $\beta_j$, $\gamma_{jk}$ are parameters. Based on the assumption of expenditure-minimizing behaviour on the part of households, a system of share equations describing residential demands for the various energy sources is derived from (5) using Shephard's lemma and equating $E_t\ (u_t, p_t)$ with observed per-household energy expenditures.

Applying the logarithmic form of Shephard's lemma, the expenditure share equation for the ith good is obtained as the derivative of $\ln C$ with respect to the logarithm of the ith price, $\ln p_i$. Equating $E_t\ (u_t, p_t)$ with observed total expenditure on the group of goods in question, such as per-household energy expenditures, .the following system of expenditure share equations is obtained:

(6) $$s_{it} = \alpha_i + \sum_k \gamma_{ik} \ln p_{kt} + \beta_i \ln[E_t / P_t]$$

where:

$s_{it}$ is the expenditure share of the $i^{\text{th}}$ energy source in period $t$ ,

$p_{kt}$ is the price of the $k^{\text{th}}$ energy source in period $t$ ,

$E_t$ is the observed per-household expenditure on residential energy in period $t$, and

(7) $$\ln P_t = \alpha_0 + \sum_k \alpha_k \ln p_{kt} + \tfrac{1}{2} \sum_j \sum_k \gamma_{jk} \ln p_{jt} \ln p_{kt} \; .$$

In practice, the parameter $\alpha_0$ can be difficult to estimate, and is sometimes set to zero, or to the minimum level of expenditure in the sample. An alternative formulation, which simplifies the empirical analysis by avoiding both the difficulty of empirically identifying $\alpha_0$, and the estimation difficulties associated with the nonlinear specification that results when (7) is substituted into (6), involves replacing the nonlinear price index (7) with the Stone price index, $\ln P_t^* = \sum_k s_{kt} \ln p_{kt}$ , where $s_{kt}$ is the expenditure share for the $k^{\text{th}}$ energy source in period $t$. This yields the popular Linear Approximation to the Almost Ideal Demand System (or LAIDS), which has been estimated frequently in empirical demand applications (Buse, 1994).

It is often the case in studies of residential energy demand that there is a need to include additional variables in the specification. Specifically, while the expenditure function from which the demand (share) equations are derived is a function only of the relevant prices and total expenditure, it may be conditioned on a number of other factors. For example, in a residential context, energy demands are known to be dependent on weather conditions, since these affect the need for space heating and cooling, which are typically the major end uses in the residential sector. The extent of weather-induced need for space heating and cooling (*e.g.*, Dunstan and Schmidt, 1988) can be incorporated in the model by including heating degree-days (hdd) and cooling degree-days (cdd) as additional explanatory variables. In the specification in (6), with $\ln P_t$ replaced by $\ln P_t^*$, this is equivalent to specifying the parameter $\alpha_i$ as a linear function of (the logarithms of) these variables:[2]

(8) $$\alpha_i = \alpha_i^* + c_i \ln hdd_t + d_i \ln cdd_t ,$$

where $c_i$ and $d_i$ (i=1,…,n) are additional parameters to be estimated. Other conditioning variables may also be included in the share equations by modifying (8). However, in some cases there are other alternatives. For example, in a residential context it might be expected that energy demand by a household would increase with household size. While household size could be included as an additional variable in (8), and hence in the share equations, the number of parameters to be estimated increases by n for every variable added. In the case of household size this is sometimes avoided by defining the quantity of energy and expenditure in per-capita terms by dividing household values for these variables by household size.

With all these modifications incorporated, the LAIDS share equations have the following linear form:

(9) $$s_{it} = \alpha_i + \sum_k \gamma_{ik} \ln p_{kt} + \beta_i \ln[E_t / P_t^*] + c_i \ln hdd_t + d_i \ln cdd_t ,$$

where in addition to the previous definitions,

$P_t^*$ is the Stone price index, defined as: $\ln P_t^* = \sum_k s_{kt} \ln p_{kt}$ ;

$hdd_t$ is heating degree-days in period *t* (for example, degree-days below $18^{\circ}$ Celsius),

[2] There is no real need to take logarithms of *hdd* and *cdd*, even though other explanatory variables appear in logarithmic form. These variables can often take quite large values, and the use of logarithms reduces the scale.

$cdd_t$ is cooling-degree-days in period *t* (for example, degree-days above 18° Celsius).

Another consideration with the specification of these expenditure share equations concerns dynamics. As noted in the single-equation energy demand models, energy demands do not always respond instantaneously to changes in prices for a variety of reasons, and this is typically captured by including lagged energy consumption terms in the single-equation specification. A similar approach can be used with a system of expenditure share equations, although it is more common in this case to include lagged expenditure shares. Specifically, expenditure shares in the current period are assumed to adjust only partially to their desired level from the previous (last period) level:[3]

(10) $$s_t - s_{t-1} = \boldsymbol{\Lambda}^* (s_t^* - s_{t-1})$$

where $s_t = (s_{1t}, \ldots, s_{nt})'$ is a vector of the expenditure shares of the *n* different energy sources in period *t*, $s_t^*$ is a vector of desired shares derived from expenditure-minimizing behaviour (for example, as specified in (24)), and $\boldsymbol{\Lambda}^*$ is an (*n* x *n*) matrix of adjustment coefficients with $\lambda_{ij}^*$ being the (i,j) element. In the simplest form of this specification $\boldsymbol{\Lambda}^*$ is a constant, diagonal matrix, so that the adjustment for each energy source depends only on its own desired and previous levels:

(11) $$s_{it} - s_{it-1} = \lambda_{ii}^* (s_{it}^* - s_{it-1})$$

Rearranging (11) yields:

(12) $$s_{it} = \lambda_{ii}^* s_{it}^* + (1 - \lambda_{ii}^*) s_{it-1},$$

where $s_{it}^*$ is given by the right-hand side of (9). Since all the terms on the right-hand side of (9) involve parameters, $\lambda_{ii}^* s_{it}^*$ just means that all the parameters in (9) are multiplied by $\lambda_{ii}^*$. In practice, this need not be explicitly incorporated in the equation, as whether the parameters that

[3] A number of criticisms have been leveled at the use of partial adjustment formulations in energy demand analysis. Berndt *et al.* (1981) indicate that partial adjustment mechanisms are not based on optimizing behaviour, and that the resulting estimated long-run elasticities do not necessarily exceed their corresponding short-run values. Instead, they propose a model with endogenous adjustment. Hogan (1989) also identifies a potential misspecification that results when the partial adjustment process is expressed in terms of expenditure shares rather than quantities. In the alternative specification that Hogan (1989) suggests, the coefficients of the lagged shares are themselves functions of prices.

are estimated are $\alpha_i$, $\beta_i$, $\gamma_{ik}$, etc., or $\alpha_i^* = \lambda_{ii}^*\alpha_i$, $\beta_i^* = \lambda_{ii}^*\beta_i$, $\gamma_{ik}^* = \lambda_{ii}^*\gamma_{ik}$, etc., makes no difference to the estimation. Thus, defining $\lambda_i = (1-\lambda_{ii}^*)$ for notational convenience, the expenditure share equations for the LAIDS model incorporating the dynamic specification in (12) have the form:

(13) $$s_{it} = \alpha_i + \sum_k \gamma_{ik} \ln p_{kt} + \beta_i \ln[E_t / P_t^*] + c_i \ln hdd_t + d_i \ln cdd_t + \lambda_i s_{it-1},$$

whereas if the full specification in (10) is used rather than (12), then the last term in (13) would be replaced with $\sum_j \lambda_{ij} s_{jt-1}$, where $\lambda_{ij} = (1-\lambda_{ij}^*)$. Note that in this latter case, since the lagged shares sum to unity, only (n-1) of the n lagged shares can be included in each equation.

When estimating the equation system in (13), adding-up of the share equations (that is, the requirement that the shares sum to unity) requires the following parameter restrictions:

(14) $\lambda_i = \lambda$, $i$=1, …, $n$,

(15) $\sum_i \alpha_i + \lambda = 1$,

(16) $\sum_i \beta_i = \sum_i c_i = \sum_i d_i = 0$, and

(17) $\sum_i \gamma_{ik} = 0, \quad k = 1, \ldots, n$.

The first of three parametric restrictions requires that in (13), the lagged own-share in each equation have the same coefficient. In other words, the speed of adjustment is restricted to be the same across all energy sources. This restriction arises because the sum of the lagged shares is unity, so that $\sum_{i=1}^{n} \lambda_i s_{it-1} = \sum_{i=1}^{n-1} (\lambda_i - \lambda_n) s_{it-1} + \lambda_n$, and for this to be constant in different time periods (observations), it must be independent of the shares, which can only occur if (14) holds. As a result of (14), $\sum_{i=1}^{n} \lambda_i s_{it-1} = \lambda$, so that the sum of the shares adding to 1 now results in the restriction in (15). The restriction that the lagged share in each equation must have the same coefficient can be avoided by using (10) and therefore including (n-1) of the lagged shares in each equation by replacing $\lambda_i s_{it-1}$ in (13) by $\sum_{j=1}^{n-1} \lambda_{ij} s_{jt-1}$, in which case the parameter restrictions in (14) and (15) are replaced by:

(14.1) $\sum_i \lambda_{ij} = 0,\ j = 1, \dots, (n-1)$, and

(15.1) $\sum_i \alpha_i = 1$.

Although the prices and expenditure in share equations such as (13) are typically expressed in nominal terms, if the demands satisfy the homogeneity condition (that is, they are homogeneous of degree zero in prices and total expenditure or total cost), so that a scaling of all prices and total expenditure does not affect the quantities that are demanded, then in (13):

(18) $\sum_{k=1}^{n} \gamma_{ik} = 0$.

In this case the term $\sum_{k=1}^{n} \gamma_{ik} \ln p_{kt}$ in (13) can be rewritten as $\sum_{k=1}^{n-1} \gamma_{ik} \ln(p_{kt} / p_{nt})$. Consequently, any common price index that is used to convert nominal prices to real prices will cancel out when the price ratio terms $(p_{kt} / p_{nt})$ are calculated. Hence, with homogeneity imposed it does not matter whether real rather than nominal prices and expenditure are used. Since the homogeneity condition follows directly from the adding-up condition – that the sum of expenditures on (or costs of) each energy source equals total expenditure on (or cost of) energy – it is typically expected to hold in demand systems like (13) and would often be imposed.

A second set of conditions that would be expected to hold in demand systems like (13) are what we refer to as the standard symmetry conditions, namely:

(19) $\gamma_{ik} = \gamma_{ki}$ ($i, k = 1, \dots, n$).

These conditions are required for identification purposes, and follow from the fact that the price term in (13), $\sum_{k=1}^{n} \gamma_{ik} \ln p_{kt}$, is obtained as the derivative with respect to the logarithm of the $i^{\text{th}}$ price of a cross-product term such as $\frac{1}{2}\left(\sum_{j=1}^{n} \sum_{k=1}^{n} \gamma_{jk}^{*} \ln p_j \ln p_k\right)$ that appears in the cost or expenditure function, such as (21). Technically this derivative equals $\sum_{k=1}^{n} \frac{1}{2}(\gamma_{ik}^{*} + \gamma_{ki}^{*}) \ln p_{kt}$, but since $\gamma_{ik}^{*}$ and $\gamma_{ki}^{*}$ always appear in the additive form $(\gamma_{ik}^{*} + \gamma_{ki}^{*})/2$, neither is separately identified, so that this term is simply redefined as $\gamma_{ik}$, and by definition, $\gamma_{ik} = \gamma_{ki}$ ($i$, $k$ = 1, …, $n$). In many circumstances these standard symmetry conditions are equivalent to the conditions required for Slutsky symmetry to hold, where the Slutsky symmetry conditions are the requirement that the derivative of the compensated demand for $i^{th}$ good with respect to the $j^{th}$ price is equal to the derivative of the $j^{th}$ compensated demand with respect to the $i^{th}$ price, in other words that the second derivatives of the cost or expenditure function are the same regardless of the order in which the derivatives are taken. However, we emphasize here that (19) are just identification conditions so that they would generally be imposed on (13). For subsequent analysis it is also important to note that due to the adding-up condition (the sum of the shares summing to 1), as reflected in the parametric restrictions in (14) through (17), the imposition of the standard symmetry conditions in (19) means that the homogeneity condition (18) will automatically be satisfied.

Since the parameter estimates themselves have little direct interpretation, and interest generally centres on the estimated price responses that are determined from the parameter estimates and data. For the LAIDS model, the price elasticities for the various energy sources (using either the real or relative price specifications) can be calculated from the estimated parameters using the relationship:[4]

(20) $$\eta_{ik} = \gamma_{ik} / s_i - \beta_i \, s_k / s_i - \omega_{ik} \, ,$$

where $\omega_{ik} = 1$ if $i = k$, and $\omega_{ik} = 0$ otherwise. Income (or total expenditure) elasticities can be calculated using:

(21) $$\eta_{iE} = 1 + \beta_i \, / s_i$$

Since these elasticities depend on the shares (and should be evaluated using estimated values of these shares), they differ for each observation. A common practice is to present estimated elasticities evaluated at the sample means for the explanatory variables, but this has a number of drawbacks. First, the point where each variable equals its sample mean is not necessarily even closely related to any of the sample observations. More importantly, one of the main advantages of using flexible functional forms as opposed to as ingle-equation linear in logarithms energy demand equation is that the elasticities are not assumed to be constant, and indeed will generally vary throughout the sample. Over time, price responsiveness may be changing quite extensively,

[4] See Buse (1994) for an evaluation of the various possible elasticity expressions that can be used with the LAIDS model. The expression in (20) is the most widely used for the price elasticity and, according to Buse's results, is marginally the best. Buse finds that the income elasticity as defined in (21) to be superior to other alternatives.

but this information is lost if price elasticities are only evaluated at the sample mean. Thus, a recommended approach would be to calculate elasticities at each sample point, although they may only be presented for selected observations, such as every 10 years or matching certain periods in energy demand evolution (pre-OPEC, etc.). Of course, estimated elasticities for all observations in the sample could easily be presented using a graphical approach. Ideally, in order to evaluate the significance of the elasticities, estimated standard errors for the elasticities should be calculated using the delta method mentioned earlier, and in a graphical approach, confidence bounds for the elasticities could be included.

## 4. Non-stationarity and implications for energy demand modelling and estimation

For some time, empirical modelers have worried about the fact that when using time-series data, the variables often exhibited strong trends. As a result, when one of these variables, such as energy demand, was regressed on others, such as price and income, a strong relationship might be found just because of the trends that were present in the variables. In the late 1970s and early 1980s this led to series of papers questioning whether the results of empirical time-series estimations were actually useful, or should just be regarded as spurious (e.g., Hendry 1980). Typical evidence of such spurious regressions would be reflected in very good fit statistics, such as $R^2$ close to 1.0, and very low values of the Durbin-Watson statistic, indicating strong autocorrelation, with an autocorrelation coefficient often relatively close to 1.0. While the initial response to these concerns was to conduct extensive diagnostic testing, later a more common approach involved testing whether variable were indeed non-stationary, and to deal with identified non-stationarity through differencing of variables and estimation of short-run models or via estimation of cointegration relationships and/or error correction models. Subsequently, various other approaches were also adopted. The remainder of this section focuses on the different methodologies that have been used in the context of energy demand models and modelling.

If unit root tests reveal that certain variables are non-stationary but can be converted to stationarity by first-or higher-order differencing, that is they are integrated of order 1 (I(1))or higher, while other variables may be stationary without the need for differencing (I(0)), then these variables can be included in a model that can be estimated by standard means. For example, if energy consumption *E*, income *Y*, and energy price *P*, are all I(1), while a weather variable *W* is I(0), then a valid relationship that could be estimated is:

(22) $$\Delta \ln E_t = \beta_1 + \beta_2 \Delta \ln P_t + \beta_3 \Delta \ln Y_t + \beta_4 W_t + \varepsilon_t$$

where $\Delta \ln E_t = E_t - E_{t-1}$, and $\Delta \ln P_t$ and $\Delta \ln Y_t$ are defined analogously. In this framework, the coefficients can only be interpreted as short-run coefficients, since in the long-run all the differenced variables would equal zero so that (22) would be vacuous. An approach that has been used to deal with this issue in a number of studies involves estimation of an error correction model (ECM). Specifically, even though *E*, *Y*, and *P* may all be I(1), there may be a (one or more) cointegrating relationship among these variables such that the error term in this relationship is itself stationary. This cointegrating relationship is viewed as representing the long-run or equilibrium relationship, so that the residuals from this relationship represent the deviation from equilibrium. Since – if a cointegrating relationship exists – these residuals are I(0), their lagged value, interpreted as the deviation from equilibrium in the previous period, may be included in (22). In practice a two-step process (the Engle-Granger procedure) is frequently used, where (following unit root tests to verify the non-stationarity of the variables) OLS estimation of (1) yields estimated residuals, $\hat{e}_t$, which (after being confirmed as a stationary series) are then lagged and included as an additional regressor in (22). This yields the ECM, so named because the change in energy consumption in any period is partly in response to the error (deviation from equilibrium) in the previous period:

$$(23) \qquad \Delta \ln E_t = \beta_1 + \beta_2 \Delta \ln P_t + \beta_3 \Delta \ln Y_t + \beta_4 W_t + \beta_5 \hat{e}_{t-1} + \varepsilon_t .$$

In this case, the coefficients $\beta_2$, $\beta_3$, and $\beta_4$ would be interpreted as short-run effects, while $\beta_5$ would represent the speed of adjustment to equilibrium values. Of course, the possibility of non-instantaneous responses of the dependent variable to any of the explanatory variables may still arise, and this can be accommodated by including lagged values of these explanatory variables and/or of the dependent variable in the right-hand side of (23), such as, for example, in Silk and Joutz (1997), with the resulting model estimated using standard methods.

A possible problem with this approach arises if the sample size is small, since the unit root and cointegration tests are less reliable in these circumstances, and the resulting regression estimates are not robust (Mah, 2000). In interpreting whether a sample is small here, the relevant concern is the length of the period covered rather than just the number of observations (e.g., Hakkio and Rush, 1991). An approach which avoids these problems and has better small-sample properties is based on the autoregressive distributed lag (ARDL) framework outlined in Pesaran and Shin (1999), and applied in an energy demand context by Narayan and Smyth (2005). With this approach, the first step is to specify the ECM in (23) as an ARDL model by including lags of the dependent variable and of the potentially non-stationary explanatory variables on the right hand side. In addition, instead of estimating using a two-step process, the unobserved actual value of the lagged residual, $e_{t-1}$ is used instead of $\hat{e}_{t-1}$, where, from (1), $e_{t-1}$ can be written as:

(24) $$e_{t-1} = (\ln E_{t-1} - \beta_1 - \beta_2 \ln P_{t-1} - \beta_3 \ln Y_{t-1}).$$

This yields an ECM that has the form:

(25) $$\begin{aligned}\Delta \ln E_t = \beta_1 + \beta_{1A}\, t + \sum_{i=0}^{p} \beta_{2i} \Delta \ln P_{t-i} + \sum_{j=0}^{q} \beta_{3j} \Delta \ln Y_{t-j} + \sum_{k=1}^{r} \beta_{4k} \Delta \ln E_{t-k} + \beta_5 W_t \\ + \beta_6 \ln E_{t-1} + \beta_7 \ln P_{t-1} + \beta_8 \ln Y_{t-1} + \varepsilon_t\end{aligned} .$$

where *t* is a time trend. Note that in this case, no pre-testing is done to determine whether any of the variables *E*, *P*, and *Y* are non-stationary, or whether there is a cointegrating relationship. Rather, (25) is simply estimated and the hypothesis that $\beta_6 = \beta_7 = \beta_8 = 0$ is tested using a non-standard F test with critical values from Pesaran and Pesaran (1997). Rejection of this hypothesis reveals that there is a long-run cointegrating relationship among the variables in the original model. Once this has been confirmed, the long-run relationship is estimated as an ARDL model (the model in (3) with distributed lags on all the explanatory variables including the lagged dependent variable), and the short-run coefficients are obtained from the standard ECM in (23), using the lagged estimated errors from the estimated ARDL long-run relationships.

An alternative approach to the use of cointegration analysis, or the ECMs as specified above, is to use a structural time series model (STSM). This approach, argued by Harvey (1997) as being superior to a cointegration approach, has been applied in an energy demand context in a series of papers by Hunt and his coauthors (Hunt, Judge and Ninomiya, 2000, 2003a, 2003b; Hunt and Ninomiya, 2003, 2005; and Hunt and Al-Rabbaie, 2006). One of the main advantages of such models is that they allow for trends that are not necessarily linear and deterministic. As Harvey (1997) notes, to the extent that this is the case, analysis that begins by detrending the data by regressing variables on time will render all subsequent analysis invalid. The STSM model is also well suited to dealing with seasonality that is not deterministic and linear, that is, that is not well represented by the inclusion of seasonal dummy variables; such a model is considered by Hunt and Ninomiya (2003). A particular feature of the STSM is that a "standard" model with a linear deterministic trend and seasonal dummy variables is a special case of the STSM. For illustrative purposes we consider a STSM version of the model in (1):

(26) $$\ln E_t = \mu_t + \beta_2 \ln P_t + \beta_3 \ln Y_t + e_t,$$

where:

(27) $\mu_t = \mu_{t-1} + \gamma_{t-1} + \nu_t$, where $\nu_t \sim N(0, \sigma_\nu^2)$,

(28) $\gamma_t = \gamma_{t-1} + \omega_t$, where $\omega_t \sim N(0, \sigma_\omega^2)$,

Here (27) and (28) represent the level and slope of the trend. In the context of energy demand models, Hunt and Ninomiya (2005) refer to $\mu_t$ as the Underlying Energy Demand Trend (UEDT). If the so-called hyperparameters, $\sigma_\nu^2$ and $\sigma_\omega^2$, are both zero, then the model in (26) – (28) reverts to a model with a deterministic linear trend:

(29) $\ln E_t = \beta_1 + \gamma t + \beta_2 \ln P_t + \beta_3 \ln Y_t + e_t$.

To conclude their specification Hunt and Ninomiya (2005) allow for dynamic effects by including polynomial distributed lags on $\ln E$, $\ln P$, and $\ln Y$. Estimation of the specification in (10)-(12), modified in this way, is accomplished using Maximum Likelihood estimation, with the Kalman filter used to obtain the optimal estimates of the last-period values of the level and slope of the trend, while a smoothing algorithm of the Kalman filter is used to obtain optimal estimates of the trend components. While this approach may seem particularly complex compared to, for example, a two-step ECM, as Harvey (1997) notes, it is not necessary to understand the Kalman filter in order to conduct this analysis as the entire procedure has been implemented in a "user-friendly form" in the STAMP econometric software package.

To decide on the final specification for their STSM, Hunt and Ninomiya (2005) test down (consecutively omit lagged terms that are insignificant) from a general distributed lag formulation while ensuring that the residuals do not exhibit evidence of autocorrelation, heteroskedasticity, or non-normality. Applying their model to data for the UK and Japan, the authors found that the seasonal and trend components are indeed stochastic, that models that have a linear deterministic trend or no trend are rejected, and that the preferred model using the STSM framework was more parsimonious than using the cointegration approach, which tended to yield models that did not satisfy all the diagnostic tests in some cases.

## 5. Asymmetric demand responses to price changes[5]

Oil demand behaviour in the late 1980s provided a new set of puzzles for empirical energy researchers. During the 1970s, sharp and sustained increases in world oil prices resulted in significant reductions in the consumption of oil products in industrialized countries. For example, between 1973 and 1982, per capita use of oil products for non-transport use in the U.S. fell by 30.7%. Based on this experience, it might have been expected that the sharp and sustained decreases in world oil prices that occurred in the second half of the 1980s would be accompanied by *increases* in oil consumption. However, the response in the demand for this energy source was, at best, sluggish, with per-capita oil use for non-transport purposes in the U.S. actually falling by 3.8% in the decade that followed the sharp decreases of world oil prices in 1986.[6] These effects were generally pervasive across all sectors, and for many countries, as is demonstrated in Figure 1 which, based on the data used in Ryan and Plourde (2002), portrays the natural logarithm of real oil prices in local currency units for Canada, the US, the UK, France, and Japan, and Figure 2, which shows the natural logarithm of per-capita oil consumption in these same countries using these same data. As these figures show, while increases in the real price of oil products were accompanied by reductions in consumption until the mid-1980s, the ensuing drop in real prices did not lead to a resurgence in oil consumption levels. Indeed, during the period from the mid 1980s to 1998, per-capita oil consumption has generally continued to fall except in Japan where it has tended to remain static.

To explain this apparent asymmetric pattern of demand responses to oil price increases and decreases it is useful to recall that energy sources (or fuels) – such as electricity, natural gas, and oil products – are not of intrinsic value to consumers. Rather, they are used in conjunction with certain types of capital equipment (some of it energy-using, such as furnaces, air conditioners, motors, etc.; and some of it in the nature of a substitute, such as insulation) to provide energy-related services (such as hot or cold air for space heating, hot water, etc.), and it is these services that are valued by consumers. Three characteristics of the energy-using equipment are of particular interest: much of it is long-lived, much of it is fuel-specific, and its technical characteristics tend to be fixed. The fact that it is long-lived means that, once installed,

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[5] Some material in this section is based on Ryan and Plourde (2004), (2007).
[6] The data used in these calculations were taken from Tables 1.5 and 2.1 of U.S. Department of Energy (1999).

energy-using equipment tends to have a useful life that spans many years, often decades. In addition, much of this equipment can only be used in conjunction with a single, specific energy source to produce energy-related services. Finally, each type of energy-using equipment tends to embody a technology that specifies a given level of energy use per unit of services produced. The key consequence of all these characteristics is that they limit the scope available to consumers to respond to energy price changes.

A sustained period of high energy prices will encourage consumers to change the stock of energy-using capital (for example by purchasing more energy-efficient appliances) and to substitute capital for energy (for example by installing insulation). It will also encourage manufacturers to improve the energy efficiency of capital equipment, thereby reducing the quantity of energy needed to produce a given level of energy-related services. At the same time, one might also expect governments to act and modify building codes and standards applied to energy-using equipment in directions that encourage greater energy efficiency, for example. Indeed, both of these types of developments were observed in most industrialized countries following the world oil price increases of the 1970s. As the time period during which high energy prices are experienced lengthens, one would expect these types of adjustments to become more and more pervasive, with a resulting sustained fall in energy consumption (or, at least, in the consumption of the energy source whose price had risen).

Now, if such a period were followed by a sharp (and sustained) decrease in energy prices, the same factors that shaped responses to higher energy prices would come into play, and initial adjustments would focus on changes – this time, increases – in the intensity of use of the existing energy-using equipment. But, the average remaining useful life of this equipment would likely be longer than that in use at the time when the preceding price increases occurred, since those price increases would have accelerated equipment replacement rates. While the lower energy prices would clearly dampen incentives for energy-saving technological progress, realized technological gains – especially in terms of energy efficiency – would not be reversed. In addition, it seems rather unlikely that government policy initiatives – such as changes in building codes and appliance standards – aimed at achieving greater energy efficiency introduced in response to the higher energy prices would now be reversed as energy prices fell. For example, building codes and equipment standards would generally not be modified to encourage greater energy consumption in response to lower energy prices.

On this basis, the responsiveness of energy demand to a (sustained) price decrease would be expected to be relatively weaker (in absolute value terms) than to that of a (sustained) energy price increase that occurred earlier. Under some conditions, this could be observed for energy as a whole (if energy prices rose relative to those of other goods and services, for example), or for specific energy sources, or both. This is the starting point for a number of empirical assessments of the changing nature of the responsiveness of energy demand to energy price variations.[7] Given the evolution of world oil prices since the early 1970s, much of this work has focused on the demand for oil.

Early attempts to explain the observed sluggish response of oil demand were predominantly based on the models that had been developed for agricultural applications (*e.g.*, Bye, 1986; Watkins and Waverman, 1987; Gately and Rappoport, 1988; Shealy, 1990; Brown and Phillips, 1991). The approach adopted by Dargay (1992) for the first time allowed for separate identification within a single-equation framework of different responses to price increases and price decreases as well as to the maximum price. This approach was further refined by Gately (1992), who demonstrated that these three effects could be captured through a respecification of the price variable. Specifically, current price was represented as the sum of the maximum price to date, cumulative price decreases, and cumulative price increases that do not establish a new maximum. Empirical implementation of this framework allows for straightforward testing of the existence of asymmetric responses to price changes, since evidence that the coefficients of the three price component series are not the same would indicate that there are different responses to variations in these three price components.

Beginning with the simple dynamic single-equation log-linear specification in (3), where energy, $E$, is usually defined as per-capita oil consumption, the energy price, $P$, refers to the real price of oil and the income variable, $Y$, is typically expressed in real per-capita terms, the

[7] Early contributions to this literature include Bye (1986), Sweeney with Fenechel (1986), and Watkins and Waverman (1987), among others.

approach popularized by Dargay (1992) and Gately (1992) allows for asymmetric responses by replacing the logarithm of the real price term by a number of "components" that sum up to the original (logarithmic) price series. Three such component series are generated: the maximum historical values of the natural log of real prices (a non-decreasing series), *cumulative* sub-maximum recoveries in the natural log of real prices (a non-decreasing, non-negative series), and *cumulative* decreases (or cuts) in the natural log of real price (a non-increasing, non-positive) series. This data transformation process yields the following breakdown into three component series for the natural logarithm of the *real* price of oil, $\ln(rpoil_t)$:

$$(30) \qquad \ln(rpoil_t) \;=\; max(\ln(rpoil_t)) \;+\; cut(\ln(rpoil_t)) \;+\; rec(\ln(rpoil_t))$$

where:

$$max(\ln(rpoil_t)) = \max\{\ln(rpoil_1), \ln(rpoil_2), \dots, \ln(rpoil_t)\};$$

$$cut(\ln(rpoil_t)) = \sum_{m=1}^{t} \min\{0, [max(\ln(rpoil_{m\text{-}1})) - \ln(rpoil_{m\text{-}1})] - [max(\ln(rpoil_m) - \ln(rpoil_m)]\};$$

$$rec(\ln(rpoil_t)) = \sum_{m=1}^{t} \max\{0, [max(\ln(rpoil_{m\text{-}1})) - \ln(rpoil_{m\text{-}1})] - [max(\ln(rpoil_m) - \ln(rpoil_m)]\}.$$

Figure 3 shows the nature of this price decomposition for the natural logarithm of the US real price of oil based on the price series displayed in Figure 1.

Based on the decomposition in (30), $\ln(rpoil_t)$ would be replaced in the oil demand equation by the three components $max(\ln(rpoil_t))$, $cut(\ln(rpoil_t))$, and $rec(\ln(rpoil_t))$, and each of these terms would be permitted to have a different coefficient. Thus, the specification in (3) would be replaced with:

$$(31) \qquad \begin{aligned} \ln(qoilpc_t) \;=\; & \beta_1^* + \beta_2^* \ln(rgdppc_t) + \beta_{3A}^* max(\ln(rpoil_t)) + \beta_{3B}^* cut(\ln(rpoil_t)) \\ & + \beta_{3C}^* rec(\ln(rpoil_t)) + \beta_4^* \ln(qoilpc_{t-1}) + e_t^*. \end{aligned}$$

A test for symmetry of price responses then involves testing whether the coefficients on the three components of $\ln(rpoil_t)$ are the same, that is, whether:

$$(32) \qquad \beta_{3A}^* \;=\; \beta_{3B}^* \;=\; \beta_{3C}^*.$$

This approach has been implemented in a series of papers (*e.g.*, Gately, 1993a, 1993b; Hogan, 1993; Dargay and Gately, 1994, 1995; Haas and Schipper, 1998; Gately and Huntington, 2002), which have provided quantitative measures of the extent of the asymmetry that is present in demand responses in a number of different settings, including for various sectors of the economy and across different countries. However, in all cases the focus is on oil as a single fuel (or on total energy use), so that the empirical framework always utilizes a single-equation specification. Proceeding in this manner gives rise to a number of unresolved issues. In particular, the omission of any explicit allowance for inter-fuel substitution means that the effects of changes in the prices of alternative energy forms are not taken into consideration. Consequently, it is not possible to determine whether inter-fuel substitution accounts for any of the asymmetric effects detected in studies of oil demand and thus whether any such effects can be found once these substitution possibilities have been taken into account. On a similar note, the issue of whether this type of asymmetry can be identified for other energy sources is not addressed. Finally, as noted in an earlier section of this paper, the single-equation approach does not take into consideration the inter-related demands for various alternative energy sources when allowing for asymmetric responses to energy price changes.[8]

To address these limitations, Ryan and Plourde (2007) adopt a systems approach to modelling the inter-related demands for multiple energy sources. Thus, as described in an earlier section, spending on any one energy source – including oil – is seen as part of the overall pattern of energy expenditures. Possible asymmetries are captured via a generalization, introduced in Ryan and Plourde (2002), of the price decomposition popularized by Dargay and Gately. Ryan and Plourde (2007) also analyze the consequences of these decompositions on some standard properties of demand systems (homogeneity and symmetry), consider an alternative decomposition based on relative prices that largely avoids these consequences. They provide an empirical application using the LAIDS model involving three energy sources (electricity, natural gas, and oil products) with data from the residential sector for the province of Ontario (Canada) over the period from 1962 to 1994, which incorporates sub-periods with sharp and sustained

[8] Note also that Walker and Wirl (1993), and more recently, Griffin and Shulman (2005) consider an alternative explanation of the observed asymmetric responses to energy price changes, one that is explicitly based on the role of technological change. For an earlier discussion of the possible role of technology (and other factors) in giving rise to these asymmetries, see Sweeney with Fenechel (1986).

increases and decreases in world oil prices and in North American natural gas prices. Their results suggest that demands for these three energy sources were characterized by asymmetric responses to price changes, even after allowing for inter-fuel substitution.

## 6. A brief introduction and review of other aspects of energy demand modeling

In this section we briefly consider some other aspects of energy demand modeling. In particular, as opposed to just modeling the amount of energy – or of some particular source of energy such as electricity – that is consumed, perhaps for some purpose such as water heating, or space heating, the appliance choice that is required before using any energy for this purpose is also modeled. This approach is considered in Section 6.1. In Section 6.2 we consider an approach that has been used to determine energy end use for particular activities when available data only indicate aggregate energy consumption,

### 6.1 Joint Modelling of the Decision to Use and Intensity of Use

Typical studies of household energy demand tend to focus on total energy consumption without considering issue of appliance choice that necessarily precedes any decision concerning intensity of use of these appliances. The interrelationship between these two aspects of energy consumption was first addressed by Dubin and McFadden (1984), and their model and various variations of it have since been applied in a number of energy demand studies. The main underlying motivation for the Dubin and McFadden approach is that to the extent that the demand for durables and their use are related decisions by the consumer, specifications which ignore this fact will lead to biased and inconsistent estimates of price and income elasticities.

The general method used to estimate these models (Dubin and McFadden, 1984; Bernard, Bolduc, and Belanger, 1996) involves a two-step approach where the first step involves consumer choice among alternative appliances or energy sources, depending on the context.[9] Then, conditional on this choice, energy consumption is modelled. Note, however, that appliance holding decisions are treated as being contemporaneous with usage decisions. Nesbakken (2001) uses maximum likelihood techniques to estimate the two parts simultaneously, arguing that this results in more efficient estimation of the parameters and allows unique estimates for parameters that appear in both components of the model. Significantly different estimates of some parameters are found by Nesbakken using this more computationally-

[9] Often the different appliances will be defined in terms of the energy source that they use.

intensive simultaneous estimation approach. However, both approaches use models with the same structure, which we describe briefly.[10]

The first component concerns modeling of the decision among appliances or among different types of energy sources (fuels) for a particular appliance. This choice can be general and has been specified to include combination fuels, so that electricity, natural gas, and the combination of electricity and natural gas could be regarded as three different choices. Using a random utility approach, the unobserved utility, $U_{ij}$, obtained by household *i* from choice *j* is specified as:

(33) $U_{ij} = V_{ij}(p, y - r_j, z_j, s_j) + \varepsilon_{ij}$, i = 1,…,*N* households; j=1,…,*J* energy choices

where *p* is a vector of prices of the various fuels, *y* is income, $r_j$ is the annualized cost of purchasing and operating the chosen system (j) for basic needs, $z_j$ is a vector of attributes describing alternative *j*, $s_i$ is a vector of socio-economic characteristics characterizing decision-maker i, and $\varepsilon_{ij}$ is an error term. A household is assumed to choose energy type j if the utility from this alternative exceeds the utility from all other alternatives. Thus, the probability of this outcome, $P_{ij}$ can be expressed as:

(34) $P_{ij} = \Pr(V_{ij} + \varepsilon_{ij} > V_{ik} + \varepsilon_{ik}) = \Pr(\varepsilon_{ij} - \varepsilon_{ik} > V_{ik} - V_{ij})$ for all $k \neq j$

Given a particular assumption on the distribution of the error terms, this yields an explicit expression for the choice probability. A typical choice is that the $\varepsilon_i$ are independently and identically distributed with a Type I extreme value distribution, which yields probabilities having the multinomial logit (MNL) form:

(35) $P_{ij} = \exp(V_{ij}) / \sum_{k=1}^{J} \exp(V_{ik})$.

Bernard, Bolduc, and Belanger (1996) argue that the MNL form is not the preferred specification when the alternatives are interdependent and consider the computationally more challenging multinomial probit specification. With the 9 choices they have in their model, this method is, to

[10] For complete details, refer to studies such as those by Dubin and McFadden (1984), Bernard, Bolduc, and Belanger (1996), Nesbakken (2001), and Vaage (2000) which contain detailed expositions.

use their expression, "practically infeasible", so they use Simulated Maximum Likelihood in which the probabilities are replaced in a standard maximum likelihood estimation framework by smooth simulators constructed from an underlying latent variable model. Further details are contained in their study.

The second component concerns demand for energy conditional on the choice of energy type. Since energy demands can be obtained from the indirect utility function $V_{ij}$ using Roy's identity, the functional form for these demands will depend on the functional specification for $V_{ij}$.[11] Dubin and McFadden (1984) consider several forms for $V_{ij}$, ultimately preferring a specification that yields demands that are linear in prices and income. In their case they are interested in an electricity demand equation, although other studies using this methodology have focused on total energy demand.

[11] Roy's identity yields demand for energy of type j as $-(\partial V_{ij} / \partial p_j)/(\partial V_{ij} / \partial y)$, where $p_j$ is the price of energy type j.

## 6.2 Conditional Demand Analysis

It is often important to have information about energy consumption by end-use in a residential context. This type of information is valuable for modeling and forecasting residential energy demand, for determining the expected value and ultimate usefulness of various energy demand side management programs, as well as for determining the likely savings associated with various minimum energy performance standards that may be enacted. Of course, knowledge of the energy consumption of different appliances would also be useful to the household itself as well as for firms planning marketing strategies for specific types of appliances. The most obvious method that could be used to determine energy consumption by end-use would involve direct metering, measuring actual energy usage by each appliance. However, due to the high cost involved, this approach has only been implemented in a limited number and type of situations, most commonly in a simulated setting rather than in the context of actual functioning households. In view of the many vagaries of consumer and household behavior, it is very difficult to generalize the findings from such studies to assess likely energy consumption by end use in specific real world circumstances.

In the absence of direct metering of each appliance, it is necessary to allocate total household energy consumption among the various end-uses using some alternative means. One such approach, introduced by Parti and Parti (1980), known as Conditional Demand Analysis (CDA), involves utilizing information on total household energy consumption and on appliance holdings by each household in a regression analysis to determine energy end-use by appliance type. Given representative data from household surveys, it is then possible to assess likely energy consumption according to end use in a particular jurisdiction. However, the success of CDA for statistically isolating end-use energy consumption depends crucially on variation in appliance ownership, and the increasing and now very high penetration rates – which approach saturation levels for many appliances such as refrigerators – make it difficult to reliably estimate energy consumption for particular end-uses. In fact, overall the empirical results obtained using CDA are often disappointing, yielding negative estimates of average energy consumption for some appliances and/or implausibly large estimates for others. As a result, various attempts have been made to rectify these problems by incorporating more information in CDA model, such as actual metering data for some appliances for some households, or diary information indicating appliance use, or multiple observations for each household such as hourly energy consumption data.

The model devised by Parti and Parti (1980) to determine end-use consumption by appliance when only total energy consumption is observed is based on the observation that the total energy consumption by household *h*, $E_h$, is comprised of the sum of energy consumption in that household by each of *N* specified appliances, $E_{ih}$ (i=1,…,N), as well as energy consumption by that household attributable to remaining (unspecified) appliances, $E_{0h}$:

(36) $$E_h = E_{0h} + \sum_{i=1}^{N} E_{ih} = \sum_{i=0}^{N} E_{ih} .$$

Of course, in any particular household, energy is only consumed by an appliance if the household actually has that appliance, so we define a dummy variable $A_{ih}$ which equals 1 if household h has appliance i, and which equals zero otherwise:

(37) $$A_{ih} = \begin{cases} 1 & \text{if household } h \text{ has appliance } i \\ 0 & \text{otherwise} \end{cases}$$

where $A_{0h}$ = 1, on the basis that all households have the unspecified appliances.

It would be expected that consumption of energy by any appliance in household *h* would depend on variables such as price, income, weather, etc. To allow for this possibility, a set of *M* +1 explanatory variables, $V_{jh}$, $j = 0, ..., M$ , is introduced, where $V_{0h} = 1$ is a constant. Parti and Parti (1980) specify that energy consumption of each appliance, including the unspecified appliances, is a linear function of these explanatory variables, so that:

(38) $$E_{ih} = \sum_{j=0}^{M} \beta_{ij} (V_{jh} A_{ih}), \quad i = 0, ..., N ,$$

where $\beta_{ij}$ are unknown parameters. Hence:

(39) $$E_h = \sum_{i=0}^{N} \sum_{j=0}^{M} \beta_{ij} (V_{jh} A_{ih})$$

Given that $A_{0h}$ = 1 and $V_{0h} = 1$, this can be written in the form:

(40)
$$\begin{aligned} E_h &= \beta_{00} + \sum_{i=1}^{N} \beta_{i0} A_{ih} + \sum_{i=0}^{N} \sum_{j=1}^{M} \beta_{ij} (V_{jh} A_{ih}) \\ &= \beta_{00} + \sum_{i=1}^{N} \beta_{i0} A_{ih} + \sum_{j=1}^{M} \beta_{0j} V_{jh} + \sum_{i=1}^{N} \sum_{j=1}^{M} \beta_{ij} (V_{jh} A_{ih}) \end{aligned} .$$

If it is assumed that (38) does not hold exactly, that is, that there is a stochastic error term appended to (38), then (39) and (40) would also have an error term appended. In this context, (40) is a straightforward linear regression model, albeit with a large number of terms (1+*N*+*M*+*M***N*), where *M* is the number of explanatory variables other than the constant that are used to model the energy consumption of each appliance, and *N* is the number of appliances for which detailed household ownership information is available. Estimation of (40) would yield estimates of the $\beta_{ij}$ parameters, and use of these estimates in place of the true parameter values in (38) would yield estimates of energy consumption by each appliance in each household.

Although there are likely to be a number of econometric issues that arise with estimation of (40), including a potentially high degree of collinearity among the explanatory variables, one of the key issues is that the parameter estimates do not have any direct interpretation, and it may often not be of particular interest to estimate energy consumption of each appliance in each household. Rather, it may be more relevant to know the average consumption of each appliance across households that actually have that appliance. This information could be obtained by manipulating the estimates obtained from (40), but Parti and Parti (1980) determined how it could be obtained directly by estimating a respecified version of (40).

Define $\overline{V}_{ij}$ as the average value of the explanatory variable $V_{jh}$ for those households that actually have appliance i. Thus, with *H* representing the total number of households:

(41)
$$\overline{V}_{ij} = \frac{\sum_{h=1}^{H} V_{jh} A_{ih}}{\sum_{h=1}^{H} A_{ih}}, \quad i = 0, ..., N; \;\; j = 1, ..., M ,$$

where $\overline{V}_{i0} = 1$. Hence:

(42) $$\beta_{ij} V_{jh} A_{ih} = \beta_{ij}(V_{jh} - \overline{V}_{ij}) A_{ih} + \beta_{ij} \overline{V}_{ij} A_{ih}$$

so that in (40),

(43) $$\sum_{i=0}^{N}\sum_{j=1}^{M} \beta_{ij}(V_{jh} A_{ih}) = \sum_{i=0}^{N}\sum_{j=1}^{M} \beta_{ij}(V_{jh} - \overline{V}_{ij}) A_{ih} + \sum_{i=0}^{N}\sum_{j=1}^{M} \beta_{ij} \overline{V}_{ij} A_{ih}$$

Next, define the average amount of energy consumed by appliance i in households with this appliance as:

(44) $$\overline{E}_i = \frac{\sum_{h=1}^{H} E_{ih} A_{ih}}{\sum_{h=1}^{H} A_{ih}} = \frac{\sum_{h=1}^{H} E_{ih}}{\sum_{h=1}^{H} A_{ih}} , \quad i = 0, ..., N .$$

Using (38) and (41), this can be rewritten as:

(45) $$\overline{E}_i = \frac{\sum_{h=1}^{H}\sum_{j=0}^{M} \beta_{ij} V_{jh} A_{ih}}{\sum_{h=1}^{H} A_{ih}} = \sum_{j=0}^{M} \beta_{ij} \overline{V}_{ij} , \quad i = 0, ..., N$$

Now since:

(46) $$\overline{E}_i A_{ih} = \sum_{j=0}^{M} \beta_{ij} \overline{V}_{ij} A_{ih} = \beta_{i0} A_{ih} + \sum_{j=1}^{M} \beta_{ij} \overline{V}_{ij} A_{ih}$$

it follows that:

(47) $$\sum_{i=0}^{N} \overline{E}_i A_{ih} = \sum_{i=0}^{N} \beta_{i0} A_{ih} + \sum_{i=0}^{N}\sum_{j=1}^{M} \beta_{ij} \overline{V}_{ij} A_{ih}$$

Therefore,

(48) $$\sum_{i=0}^{N} \overline{E}_i A_{ih} - \sum_{i=0}^{N}\sum_{j=1}^{M} \beta_{ij} \overline{V}_{ij} A_{ih} = \beta_{00} + \sum_{i=1}^{N} \beta_{i0} A_{ih}$$

Substituting (48) and (43) into (40) now yields:

(49) $$\begin{aligned} E_h &= \sum_{i=0}^{N} \overline{E}_i A_{ih} + \sum_{i=0}^{N}\sum_{j=1}^{M} \beta_{ij}(V_{jh} - \overline{V}_{ij}) A_{ih} \\ &= \overline{E}_0 + \sum_{i=1}^{N} \overline{E}_i A_{ih} + \sum_{j=1}^{M} \beta_{0j}(V_{jh} - \overline{V}_{0j}) + \sum_{i=1}^{N}\sum_{j=1}^{M} \beta_{ij}(V_{jh} - \overline{V}_{ij}) A_{ih} \end{aligned}$$

With an error term appended, this is again in the form of a linear regression model, where the explanatory variables are the appliance ownership dummy variables ($A_{ih}$), the variables $(V_{jh} - \overline{V}_{0j})$ which reflect the difference between the value of the j^th^ explanatory variable for the h^th^ household ($V_{jh}$) and its average value over all households ($\overline{V}_{0j}$), and interaction terms $(V_{jh} - \overline{V}_{ij})A_{ih}$ which for each appliance type i, are either the difference between the value of the j^th^ explanatory variable for the h^th^ household ($V_{jh}$) and its average value over all households with that appliance ($\overline{V}_{ij}$), or zero if the household does not have that particular appliance.

A particularly attractive feature of (14) is that the estimated coefficients on the appliance ownership dummy variables are estimates of $\overline{E}_i$, the average consumption of energy attributable to each specified appliance (for households that have that appliance), while the intercept is an estimate of $\overline{E}_0$, which is average energy consumption by unspecified appliances that are owned by all households. Since these measures are all estimated directly, estimated standard errors are obtained directly and confidence intervals can be readily calculated.

Empirically, there are a number of problems with estimation of (14). First, there are just as many coefficients in (14) as in the original specification in (5), and although multicollinearity may have been alleviated to some degree by the respecification, in practice it is generally a major problem with either model formulation. The model requires some households to have certain appliances and not others, so that $A_{ih}$ is not a constant, but as penetration rates for any appliance approach saturation, estimation will become problematic. Of course, if *all* households have a particular appliance, this appliance would simply become part of the set of unspecified appliances, and specific end-use energy consumption for that appliance would not be estimated separately.

Perhaps the most important consideration with estimation of (14) is that the interpretation of the coefficients only makes sense if the estimated coefficients on the appliance ownership dummy variables are positive. Typically, this is not the case. In the Parti and Parti (1980) application there are relatively few negative coefficients, although this may – at least in part – stem from a number of parametric restrictions that they introduce.

A number of studies have utilized the CDA framework and/or have modified it in various ways in an attempt to deal with some of the empirical problems, such as negative coefficients on the appliance dummy variables, which taken at face value would indicate that compared to not using them, on average use of these appliances saves energy.

Bartels and Fiebig (1990) note that CDA is an indirect approach to the estimation of end-use loads and that problems of negative or technically implausible large estimates arise primarily because ownership of appliances among households in the sample is not sufficiently heterogeneous. The natural solution to this is to supplement the analysis with various additional sources of information. Aigner, Sorooshian and Kerwin (1984) jointly estimate CDA equations for each of the 24 hours in a day, while Caves et al (1987) propose a Bayesian approach for combining end-use profiles produced by engineering models with CDA. Larsen and Nesbakken (2002) compare the numerical results from engineering models of household end use energy consumption and CDA, and find that both have drawbacks, although they note that those associated with the engineering approach would be difficult to eliminate. As Bartels and Fiebig (1990) note, a problem with engineering models is that they ignore individual behaviour which would be expected to be important in the context of household energy use.

An alternative method of incorporating additional information into the analysis is to combine direct metering data, gathered on a selection of appliances for a sub-sample of households used in the CDA analysis, with the CDA analysis itself – Bartels and Fiebig (1996) even analyze which particular end uses (appliances) should be individually metered. The approach used by Bartels and Fiebig (1990) to combine direct metering data and CDA follows from the reformulation of the CDA model into a random coefficients framework, as developed in Fiebig, Bartels, and Aigner (1991). As these latter authors note, an additional byproduct of the random coefficients approach is that it provides for a structure for the heteroskedasticity that is frequently observed in CDA studies. In their application they also have available diary information for a subset of households that provides frequency-of-use information for a number of appliances which can be incorporated in the CDA to improve precision. Unfortunately, in many cases neither partial metering data, nor diary information, nor hourly household energy consumption data, is available.

In a Canadian context, Lacroix (2004) – using separate survey data for 1989, 1994, and 1999 – and Bernard and Lacroix (2005) apply CDA to data from Quebec and again find evidence of large and in some cases negative estimated coefficients. They investigate whether there is evidence of heteroskedasticity – which they find – as well as whether the model is too restrictive in its imposition of uniform electricity consumption among households for the same use. Using 1999 data, Lacroix and Bernard (2005) find that the uniformity hypothesis is not rejected except for electric water heating, a result they attribute to the very high saturation rate of electric water heating for households that use electricity as their main source of energy for space heating.

Ryan and Liu (2006) modify the CDA model to incorporate additional information concerning the scale of use of particular appliances by different households. These scale variables, which reflect such factors as the size of the house and of the household, as well as the capacity of certain appliances, differ from appliance to appliance. However, the inclusion of these scale variables does not appear to resolve the problems previously identified in empirical applications of CDA, with average electricity consumption found to be negative for several appliances. Some possible solutions are investigated, including grouping of appliances so that they have a common coefficient, but this is only found to help in some cases, and tests of the various groupings indicate that not all are appropriate. Alternative methods of dealing with the empirical problems encountered with CDA models require further investigation.

## 7. Summary and Conclusion

In this paper we have reviewed energy demand models and modelling in the context of the historical evolution of the approaches that have been used. The literature on this topic has blossomed and it is not possible to consider all of the many different models and modelling approaches that have been used. Rather, we have attempted to explain the key features of the models and estimation methodology – which tend to be inextricably linked – as well as how these have evolved over time and, where possible, suggesting at least partial explanations for this evolution.

Our review helps to demonstrate that there is no single "right" approach to modeling energy demand. Models differ according to various circumstances, and the model that might be applicable in one setting may be totally inappropriate in another. While the development of some approaches appears to have ended, in some cases this may reflect the mathematical complexity that was involved. In others, it may reflect a waning interest in the particular questions that those approaches were designed to answer. As the particular aspects of energy demand that are of interest continue to change over time, some of these previously discontinued approaches may be resurrected, particularly as data availability continues to improve along with econometric software that enables practitioners to more easily estimate increasingly more complex model specifications.

**Figure 1: Natural Logarithm of Real Oil Price (local currency)**

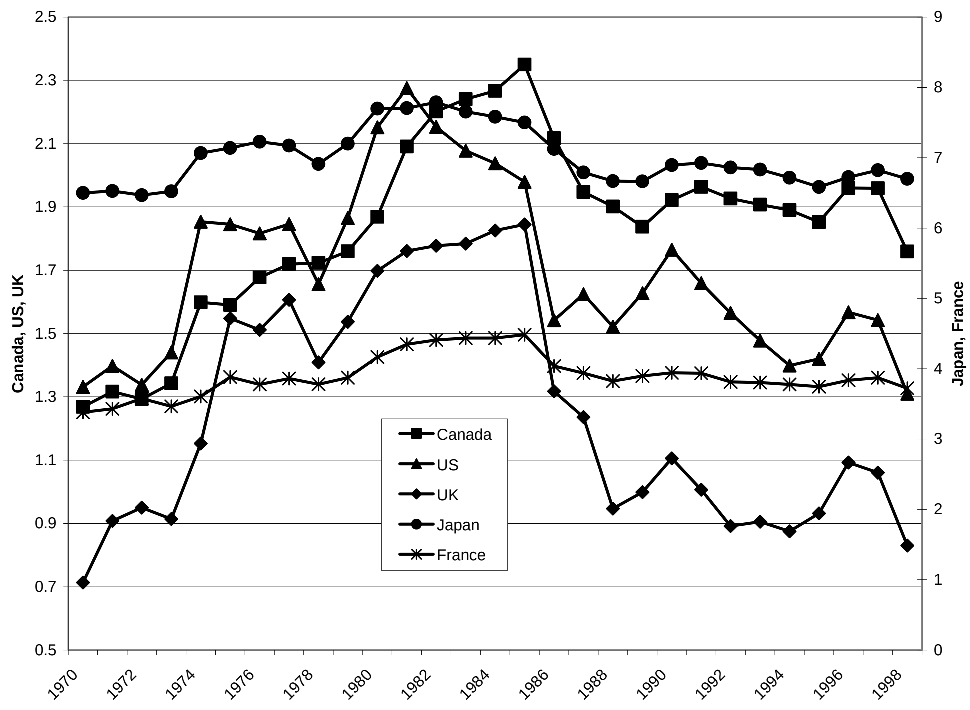



**Figure 2: Natural Logarithm of Per-Capita Oil Consumption**

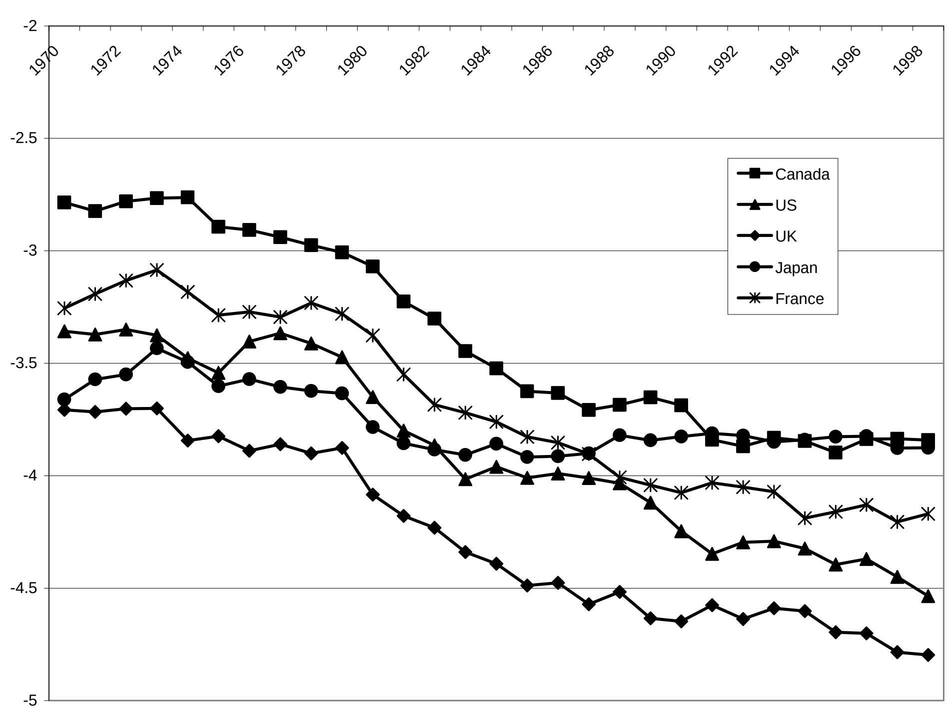
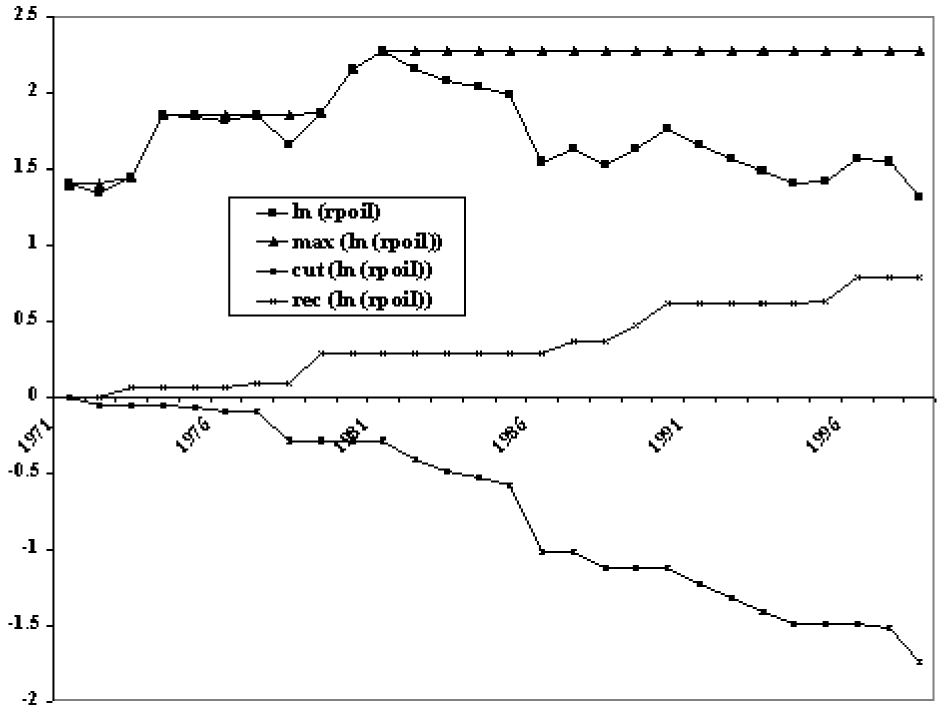


**Figure 3: Decomposition of Natural Logarithm of U.S. Real Oil Price**



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